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ONE-DIMENSIONAL MODELLING OF PHASE HOLDUPS IN THREE-PHASE STRATIFIED FLOW

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Abstract—This paper reports an investigation of the application of the three-fluid model to estimate phase holdups in three-phase stratified flow. A computer code (PRESBAL) was developed and used to apply the three-fluid model with a variety of assumptions regarding modelling of wall and interface shear stresses. Alternative definitions of equivalent diameters were also investigated. Experimental data for oil–water–air stratified flow obtained from the high pressure multiphase flow WASP facility and the data published by others have been chosen to compare with the various predictions. Tentative recommendations are made on the choice of friction relationships to provide the best representation of the data. © 1997 Elsevier Science Ltd.

Key Words: equivalent diameter, friction factor, one-dimensional, holdup, stratified, three-phase, shear stress

1. INTRODUCTION

Three-phase (oil–water–gas) flow is important in a wide variety of applications in hydrocarbon recovery. The overall objective of the study of which the work reported here forms part is to develop generic prediction methods for a specific form of such flows, namely three-phase stratified flows. Though there have been reasonably extensive studies of two-phase (gas–liquid) stratified flow in both tubes and rectangular channels, three-phase stratified flow has met relatively little attention.

For two-phase stratified flow, the most widely used modelling approach is to start with the simplified one-dimensional phasic momentum balance equations. For steady incompressible flow in a horizontal pipe, these are in the form:

$$-A_G\left(\frac{dp}{dz}\right) - \tau_G S_G - \tau_i S_i = 0 \qquad [1]$$

$$-A_L\left(\frac{dp}{dz}\right) - \tau_L S_L - \tau_i S_i = 0, \qquad [2]$$

where A_G and A_L are the cross sections occupied by the gas and liquid, respectively, (dp/dz) is the pressure gradient, τ_G, τ_L and τ_i are the averaged gas–wall, liquid–wall and gas–liquid interfacial shear stresses and S_G, S_L and S_i are the gas–wall, liquid–wall and gas–liquid interfacial perimeters. These forms of equations were used by Agrawal *et al.* (1973) and later by Taitel and Dukler (1976); they have been the framework for most one-dimensional models. Two-dimensional models have been developed by Shoham and Taitel (1984), Issa (1988) and Hall (1992) and others but are rarely used in practical calculations. It is a simple matter to extend the one-dimensional methodology to three phase stratified flow and the resultant equations are:

$$-A_G\left(\frac{dp}{dz}\right) - \tau_G S_G - \tau_{Go} S_{Go} = 0 \qquad [3]$$

$$-A_o\left(\frac{dp}{dz}\right) - \tau_o S_o + \tau_{Go} S_{Go} - \tau_{ow} S_{ow} = 0 \quad [4]$$

$$-A_w\left(\frac{dp}{dz}\right) - \tau_w S_w + \tau_{ow} S_{ow} = 0, \quad [5]$$

where the subscripts o and w refer to the oil and water phases, respectively, and the subscript Go and ow refer to the gas–oil and oil–water interfaces, respectively. Prediction methods for three-phase stratified flow based on [3]–[5] have been developed by Hall (1992), Taitel *et al.* (1995) and Roberts (1996). If a three phase stratified flow with flat interfaces is assumed, then A_G, A_o, A_w, S_G, S_o, S_w, S_{Go} and S_{ow} are related by purely geometric relationships to the oil, water and gas holdups (volume fractions) ϵ_o, ϵ_w and ϵ_G, respectively. The various models differ in the choice of equations used for calculating the shear stresses (τ_G, τ_{Go}, τ_o, τ_{ow}, and τ_w) and in the method of solution. It should be stressed that there are fundamental problems in trying to represent what is at least a two-dimensional flow using a set of one-dimensional equations. Even for a simple stratified two-phase laminar flow with a smooth interface, significant deviations may occur from the commonly applied one-dimensional flow methodology (Hall 1992; Hall and Hewitt 1993). For turbulent flows and for wavy interfaces, these problems may be exacerbated. Nevertheless, the one-dimensional flow methodology is the standard one used in the nuclear, petroleum and other industries. It can be argued that a more complex representation of the flows is currently unfeasible in any general sense and that the one-dimensional approach is the best one available. It is in this spirit that the investigations described here was carried out.

The choice of shear stress relationships is reviewed in section 2 below. The solution method adopted by Hall (1992) and Taitel *et al.* (1995) was to reduce [3]–[5] to two simultaneous equations by eliminating the (equal) pressure gradients. This mirrors the approach used by Taitel and Dukler (1976) for two-phase flow, where [1] and [2] are combined and yield a single equation. In both the two-phase and three-phase cases, the combined equations are converted to dimensionless form by introducing simplified expressions for the shear stresses; the equations are then solved iteratively. However, if the expressions for shear stresses become more complex, then this approach is unsuitable. Roberts (1996) proceeded by embodying the equations into the framework of the AEA Technology PLAC (PipeLine Analysis Code) computer program. In the work described here, a different solution approach is followed; using selected correlations for shear stresses, the water and oil levels were systematically adjusted until the pressure gradients for the three phases were equal, the final values of the liquid levels giving the phase holdups as required. This methodology has the advantage of being able to easily accommodate a whole variety of shear stress relationships; a fuller description of the procedure (embodied in a code which was given the name PRESBAL) is given in section 3.

The specific objective of the work described here was to compare predictions for three-phase stratified flow (using a variety of options for the calculations of the shear stresses) with holdup data obtained recently at Imperial College, London (Khor *et al.* 1996a) and with earlier data obtained by Sobocinski (1955). The results of these comparisons are given in section 4 below and the conclusions drawn from them are reviewed in section 5.

2. SHEAR STRESS RELATIONSHIPS

As was stated above, the crucial issue in applying the one-dimensional momentum equations to the prediction of holdups in two-phase and three-phase stratified flows is that of identifying appropriate relationships for the shear stresses. Expressions are needed for the wall shear stresses, for the gas–liquid (gas–oil) interfacial shear stress and for the liquid–liquid (oil–water) interfacial shear stress.

2.1. *Wall shear stresses*

The most common practice for calculating wall shear stresses has been to use a friction factor relationship

$$\tau_k = \tfrac{1}{2} f_k \rho_k u_k^2, \tag{6}$$

where the subscript k indicates gas, oil or water. f_k is commonly estimated from standard single phase friction factor relationships; Taitel and Dukler (1976) (and many others) use the standard Blasius (1913) form

$$f_k = 0.046\ \mathrm{Re}_k^{-0.2} \tag{7}$$

for turbulent flows and $f_k = 16/\mathrm{Re}_k$ for laminar flows. The phase Reynolds number Re_k is defined as:

$$\mathrm{Re}_k = \frac{u_k D_k \rho_k}{\eta_k}, \tag{8}$$

where ρ_k and η_k are the phase density and viscosity, respectively, and D_k is the mean hydraulic diameter of the region occupied by the phase. For rough pipes, the (implicit) Colebrook (1939) equation is often used

$$\frac{1}{\sqrt{f_k}} = 3.48 - 4.0 \log_{10}\left[\frac{2k_s}{D} + \frac{9.35}{\mathrm{Re}_k \sqrt{f_k}}\right], \tag{9}$$

where k_s is the pipe wall roughness. Alternative explicit fit to [9] (Eck, 1973) can also be applied

$$f_k = \frac{0.0625}{\left[\log_{10}\left\{\frac{15}{\mathrm{Re}_G} + \frac{\frac{k_s}{D}}{3.715}\right\}\right]^2}. \tag{10}$$

The definition of mean hydraulic diameter D_k raises problems in whether the phase-to-phase interface does or does not form part of the phase wetted perimeter, S_k in the calculation of D_k from

$$D_k = \frac{4A_k}{S_k}, \tag{11}$$

where A_k is the phase cross sectional area. For gas–liquid stratified flows, the following definitions of D_k are often used (see for instance Taitel and Dukler 1976):

$$D_G = \frac{4A_G}{S_G + S_{GL}} \tag{12}$$

$$D_L = \frac{4A_L}{S_L}, \tag{13}$$

where A_L and S_L are the cross sectional area and the perimeter of the liquid phase, respectively, and S_{GL} is the length of the gas–liquid interface. For three phase flow, Hall (1992) and Taitel *et al.* (1995) have used the definitions

$$D_G = \frac{4A_G}{S_G + S_{Go}} \tag{14}$$

$$D_o = \frac{4A_o}{S_o} \tag{15}$$

$$D_w = \frac{4A_w}{S_w}. \tag{16}$$

Comparisons with data have shown that, though the above relationships for the gas–wall shear stresses are in reasonable agreement, the predictions for liquid–wall shear stresses show considerable deviations. This has led a number of authors to propose modified correlations (Andritsos and Hanratty 1987; Kowalski 1987; Hart *et al.* 1989; Hand 1991; Srichai 1994). Both these modified correlations and the single-phase flow type correlations discussed above have been investigated in the present study.

2.2. *Gas–liquid shear stress*

The calculation of the gas–liquid interfacial shear stress is particularly difficult because of the highly disturbed nature of the interface. The friction factor is, therefore, affected by the complex interactions between the phases. Some models (for instance that of Taitel and Dukler 1976) assume that the interface is essentially flat and apply single phase relationships, i.e. Taitel and Dukler assume $f_{GL} = f_G$, and calculate the latter from [7] for turbulent flow and from $f_{GL} = f_G = 16/\mathrm{Re}_G$ for laminar flow. However, this approach is clearly inadequate and the influence of interfacial waves needs to be taken into account.

An early investigation of gas–liquid interfacial interactions was that of Hanratty and Engen (1957). Later, Linehan (1968) correlated the gas–liquid interfacial friction factor to the Reynolds number of the liquid phase, and this approach was also used by Tsiklauri *et al.* (1979) and Kim *et al.* (1985). Cheremisinoff and Davies (1979) proposed a model for turbulent–turbulent stratified flow in pipelines with interfacial friction factors evaluated using the suggestions of Cohen and Hanratty (1968) and Miya (1970) for three-dimensional small amplitude waves and roll waves respectively. In their model, the stratified smooth-stratified wavy transition criterion developed by Taitel and Dukler (1976) was used as the transition criterion between the three-dimensional wave and the roll wave regimes. However, Lee and Bankoff (1983) suggested that the transition between three-dimensional waves and roll waves could be determined from the critical gas phase Reynolds number.

Sinai (1983) developed a method to evaluate the interfacial roughness in a gas–liquid stratified flow by adopting the Charnock (1955) relationship for air–sea interfacial friction. Laurinat *et al.* (1985) related the interfacial value to that of gas–wall friction factor and liquid holdup. Shoham and Taitel (1984) applied a simple solution by treating the interfacial friction factor as having a constant value of 0.0142 for the entire stratified flow regime. A similar approach was utilised by Taitel *et al.* (1995) for the evaluation of oil–water and air–oil interfacial friction factors in three-phase flow. Hamersma and Hart (1987) showed that the apparent roughness of the liquid film was related to the average value of the measured liquid film thickness in stratified flow. They applied the Colebrook (1939) equation with the calculated interfacial roughness to assess the gas–liquid interfacial friction factor. Andritsos and Hanratty (1987) claimed that the gas–liquid interfacial friction factor was strongly dependent on the ratio of the amplitude to the wavelength of interfacial waves and did not correlate well with the Reynolds number of the liquid phase as suggested by Linehan (1968); they found that large amplitude three-dimensional waves appeared once the gas superficial velocity exceeded a critical value. However, Philbin (1990) claimed that the Andritsos and Hanratty method over-predicted the value of f_{GL} at high pressures. This was perhaps to be expected since the correlation was based on an experimental study at near atmospheric pressures.

Baker *et al.* (1988) proposed a correlation for effective roughness of the gas–liquid interface in terms of Weber and viscosity numbers. This roughness was then used in the Colebrook equation [8] to calculate the interfacial friction factor. Xiao (1990) correlated the interfacial friction factor directly in terms of four dimensionless groups. Srichai (1994) developed a new empirical relationship for the interfacial friction factor which follows a similar approach to Sinai's but with a different definition of the interfacial roughness.

2.3. *Liquid–liquid shear stress*

The shear stress at the oil–water interface can also be calculated from standard single phase flow relationships and this has perhaps more justification than for the gas–oil interface since the disturbance of the interface is usually somewhat less (though inter-entrainment of the phases is in fact more likely). Hall (1992) derived the value of τ_{ow} from calculations on laminar stratified flow between flat plates. In Hall's (1992) model of three-phase stratified flow, calculations are performed for oil–gas flow from which τ_o on the bottom plate is estimated and for three-phase flow from which τ_{ow} is estimated. It is then assumed that the same ratio of τ_{ow} and τ_o would apply to the horizontal round tube data. Taitel *et al.* (1995) simplified the calculations by using a fixed value of f_{ow} ($=0.014$). An alternative approach to the calculation of τ_{ow} is to extend the correlations used for f_{Go} to the calculations of f_{ow}; for instance, if the correlations of Baker *et al.* (1988) is correct, it should also be applicable to the calculation of f_{ow}.

3. THE PRESBAL COMPUTER CODE

The code developed in the present work (PRESBAL—pressure-drop balance) is a FORTRAN 77 computer code which estimates the phase holdups by comparing the pressure drops in each phase which are derived from the momentum balances [3]–[5]. The desired solution is the point where all three phases have the same pressure gradient. The method applied is to set the total liquid height h_L and then to vary the water height h_w in steps of $0.002\ h_L$ in the range $0.05\ h_L < h_w < 0.90\ h_L$ (this range was found to cover the compared data). For each set of values of h_L and h_w, the pressure gradient for each phase were calculated using [3]–[5] using the selected relationships for the shear stresses. The differences $[(dp/dz)_G - (dp/dz)_o]$, $[(dp/dz)_G - (dp/dz)_w]$ and $[(dp/dz)_o - (dp/dz)_w]$ were calculated and the *maximum* of the three differences was determined. The optimum solution (for the given value of h_L) was that value of h_w for which the lowest value of the maximum difference between the pressure gradients was obtained. The value of h_L was changed in steps of $0.001\ D$ between $0.05\ D$ and $0.65\ D$, where D is the pipe diameter, and the procedure of stepwide change of h_w for the given h_L repeated. The solution required for the specified phase flowrate was then that pair of values of h_L and h_w for which the lowest value of the maximum difference between the calculated pressure gradients was obtained. This procedure was found to work efficiently, each calculation for the estimation of h_L and h_w taking approximately 1–2 min. The correctness of the solution was checked by entering the calculated pressure gradients and shear stresses into [3]–[5] and checking that the sums were near zero as required.

In PRESBAL, a modification of [6] was used to evaluate the fluid–wall shear stresses. Following Taitel *et al.* (1995), allowance was made for the velocities of the contacting phase, the following expressions were used to calculate the interfacial shear stresses from the friction factors

$$\tau_{Go} = \tfrac{1}{2} f_{Go}\rho_G (u_G - u_o)\,|u_G - u_o| \qquad [17]$$

$$\tau_{ow} = \tfrac{1}{2} f_{ow}\rho_o (u_o - u_w)\,|u_o - u_w|. \qquad [18]$$

The definitions of D_G and D_w used were identical to that used by Hall (1991) and Taitel *et al.* (1995) [14] and [16]. However, the definition of D_o initially used in PRESBAL [15] was found to give unphysical high values since the oil layer is often quite thin, and so the value of S_o is very low causing D_o to be large. Consequently, an under-prediction in the oil holdup will be resulted. For this reason, a modified definition for D_o was used

$$D_o = \frac{4A_o}{S_o + S_{ow}}. \qquad [19]$$

The correlations included in PRESBAL were as follows:

- Gas–wall friction. Comparisons between the use of the simple Blasius (1913) expression and the more complex Colebrook (1939) expression showed that the Blasius equation gave better agreement and it was therefore used throughout the present study.
- Oil–water and water–wall friction. Blasius (1913) [7], Colebrook (1939) [8] ($k_S = 1.52 \times 10^{-6}$ m (Sobocinski) or 4.6×10^{-5} m (WASP)), Andritsos and Hanratty (1987), Kowalski (1987), Hart *et al.* (1989), Hand (1991) and Srichai (1994).
- Gas–oil interfacial friction. Linehan (1968), Tsiklauri *et al.* (1979), Cheremisinoff and Davies (1979), Sinai (1983), Lee and Bankoff (1983), Laurinat *et al.* (1985), Kim *et al.* (1985), Andritsos and Hanratty (1987), Kowalski (1987), Baker *et al.* (1988), Hart *et al.* (1989), Hamersma and Hart (1989), Hand (1991), Xiao (1991), Hall (1992), Srichai (1994), Taitel *et al.* (1995).

- Oil–water interfacial friction. Baker *et al.* (1988) (i.e. the gas–liquid correlation applied to a liquid–liquid interface), Hall (1992) (i.e. correction to Blasius value on basis of parallel plate calculations), Taitel *et al.* (1995) (i.e. a fixed friction factor of $f_{ow} = 0.014$).

4. COMPARISONS WITH EXPERIMENTAL DATA

The comparisons were made with the early three-phase flow experimental data of Sobocinski (1955) and with the three-phase stratified flow data obtained recently from the Imperial College high pressure facility, WASP (Khor *et al.* 1996a). Sobocinski (1955) conducted his work in a horizontal test section of length 11.6 m and internal diameter 0.079 m. The oil used was diesel oil with a density of 841 kg/m^3 and viscosity of 3.83 mPas (at 24°C). The data covered a large range of flow conditions, although it had a fairly limited number of oil/water ratios and total mass flowrates. Some 17 data points were reported in the three-phase stratified and ripple regions, but only 14 of them were with phase holdup measurements. A pair of quick closing valves was used for the measurements and the measuring accuracy was not reported.

Experiments of air–oil–water stratified flows at different pressures, ranging from 0.0 to 12.0 bar(g), were conducted on the Imperial College WASP facility (Khor *et al.* 1996a) to investigate the effect of testline pressure on the flow behaviour of three-phase systems. The test section is a 37 m long, 0.0762 m internal diameter stainless steel pipe which can be inclined ±5° from the horizontal though the experiments referred to here were carried out with the pipe in a horizontal orientation. A full description of the WASP facility is given by Manolis *et al.* (1995). The oil used was Shell Tellus 22 with a density of 865 kg/m^3 and a viscosity ranging from 38 to 89 mPas dependent on the temperature. The water fraction ϵ_w and the oil fraction ϵ_o were measured using a traversing dual-energy gamma densitometer which has a measurement error of ±2% (Pan *et al.* 1993). Measurements were made automatically along a series of vertical chords across the diameter and the results processed to give average holdups for the liquid phases. A total of 156 data points have been selected for this study.

The procedure for the comparison was to select specific correlations for three of the four shear stress terms and then to calculate the values of ϵ_w and ϵ_o for each data point for the range of relationships incorporated in PRESBAL for the remaining shear stress term. In the statistical analysis, the average ratio of the predicted holdup to the measured holdup was calculated and the fractional standard deviation was determined from

$$\sigma = \sqrt{\frac{n\sum\left(\frac{\epsilon_{cal} - \epsilon_{mea}}{\epsilon_{mea}}\right)^2 - \left(\sum\frac{\epsilon_{cal} - \epsilon_{mea}}{\epsilon_{mea}}\right)^2}{n(n-1)}}, \qquad [20]$$

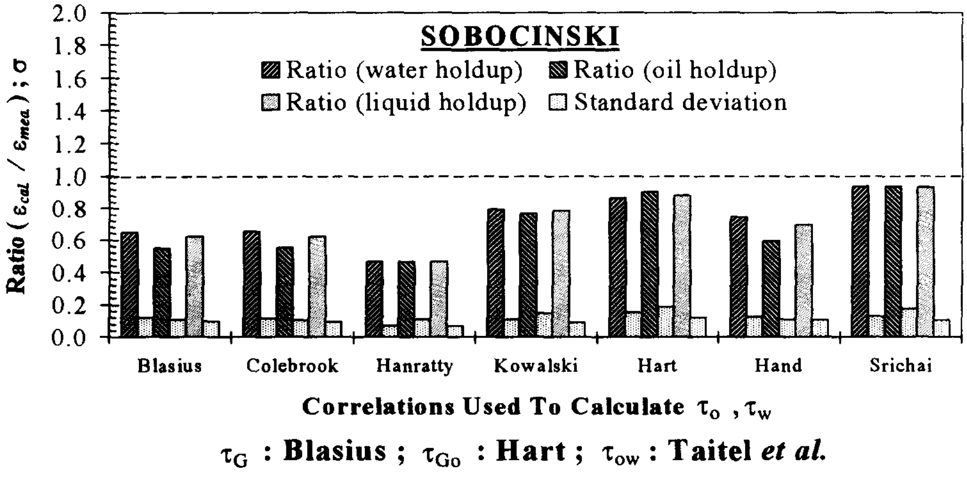

Figure 1. Comparison between correlations used to calculate τ_o and τ_w (Sobocinski data).

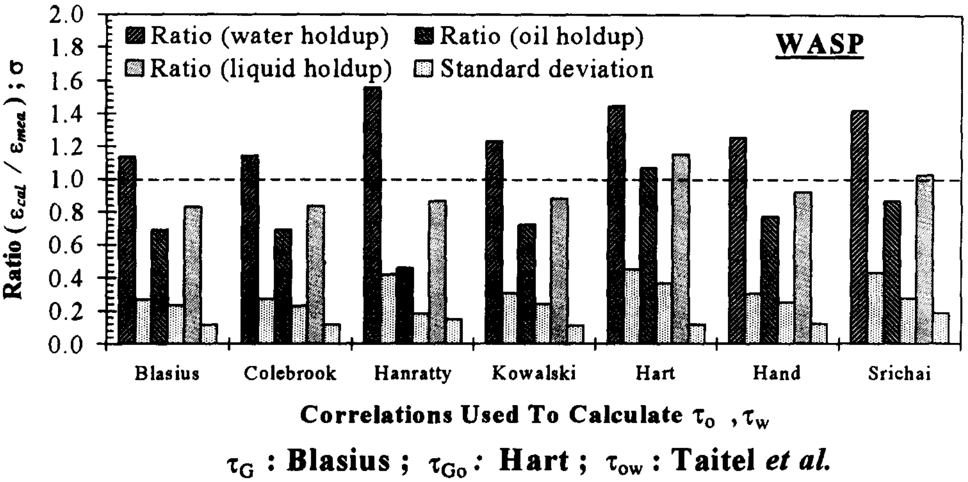

Figure 2. Comparison between correlations used to calculate τ_o and τ_w (WASP data).

where ϵ_{cal} and ϵ_{mea} are the calculated and measured values of ϵ_w or ϵ_o, respectively; and n is the total number of data points compared. As will be realised, the work carried out encompassed a simply enormous number of calculations. It is unnecessary here to present all the results in detail. Eventually, there emerged from the analysis a series of 'best' relationships for the shear stresses and for the definitions of equivalent diameters. These were

- Gas–wall shear stress (τ_G): Blasius (1913) [7]
- Oil–wall and water–wall shear stresses (τ_w and τ_o): Srichai (1994)
- Gas–oil interfacial shear stress (τ_{Go}): Hart *et al.* (1989)
- Oil–water interfacial shear stress (τ_{ow}): Taitel *et al.* (1995)
- Equivalent diameters:

$$D_G = \frac{4A_G}{S_G + S_{Go}}$$

$$D_o = \frac{4A_o}{S_o + S_{ow}}$$

$$D_w = \frac{4A_w}{S_w}.$$

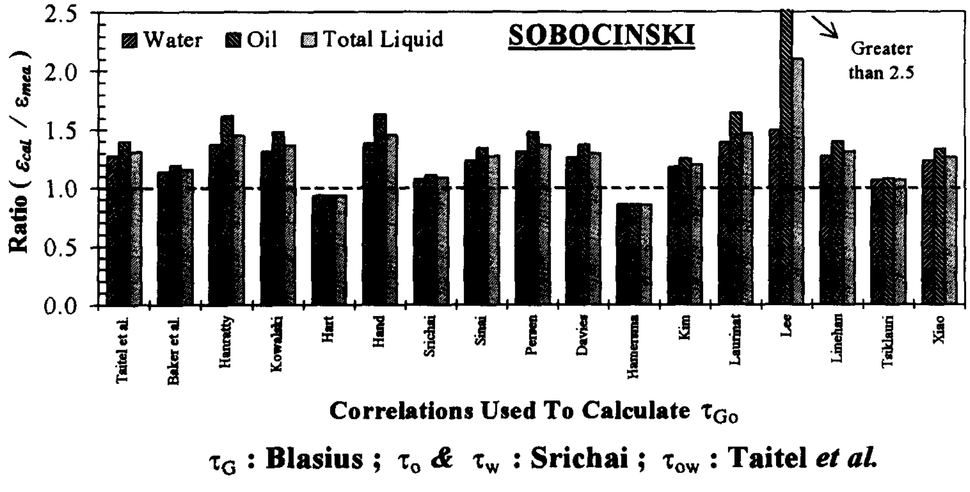

Figure 3. Comparison between correlations used to calculate τ_{Go} (Sobocinski data).

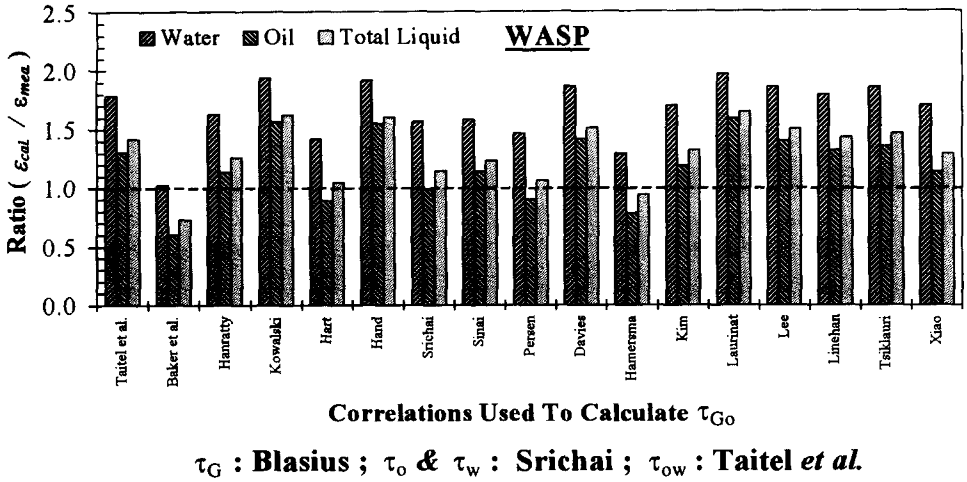

Figure 4. Comparison between correlations used to calculate τ_{Go} (WASP data).

The procedure adopted here in presenting the data is to show how the accuracy of the prediction varies with the use of other relationships for a specific quantity whilst the 'best' relationships for all the other quantities listed above are retained. The results are shown in figures 1–6; figures 1 and 2 show the comparisons of models for τ_w and τ_o. Simple models using the Blasius or Colebrook (single phase flow) approach fail badly and, of the remaining models, the model of Srichai (1994) appears to be the best. For τ_{Go} (figures 3 and 4), the overall best results were obtained using the Hart *et al.* (1989) correlation, though the Srichai (1994) correlation also performed well. Figures 5 and 6 show comparisons between models for τ_{ow}; both the Taitel *et al.* (1995) and Baker *et al.* (1988) methods worked quite well but, in view of its simplicity (i.e. f_{ow} = constant = 0.014), the Taitel *et al.* relationship is recommended.

PRESBAL can also be utilised as a tool to validate the definition of the hydraulic diameters for each phase in a three-phase stratified flow system. From the study, it is found that the predicted holdups are strongly affected by the definition of the oil phase hydraulic diameter, but are insensitive to the definitions of hydraulic diameters of the water phase and the gas phase.

In a three-phase stratified flow system, the oil layer flows co-currently between the water and the gas phases. Normally this oil layer appears to be very thin, especially for three-phase stratified flows at high pressures. The oil–wall contact length is much smaller than either the gas–oil or the oil–water wetted perimeters. Therefore, the calculated hydraulic diameter of the oil phase would be unreasonably large if only the oil–wall wetted perimeter is taken into account, i.e. the oil layer

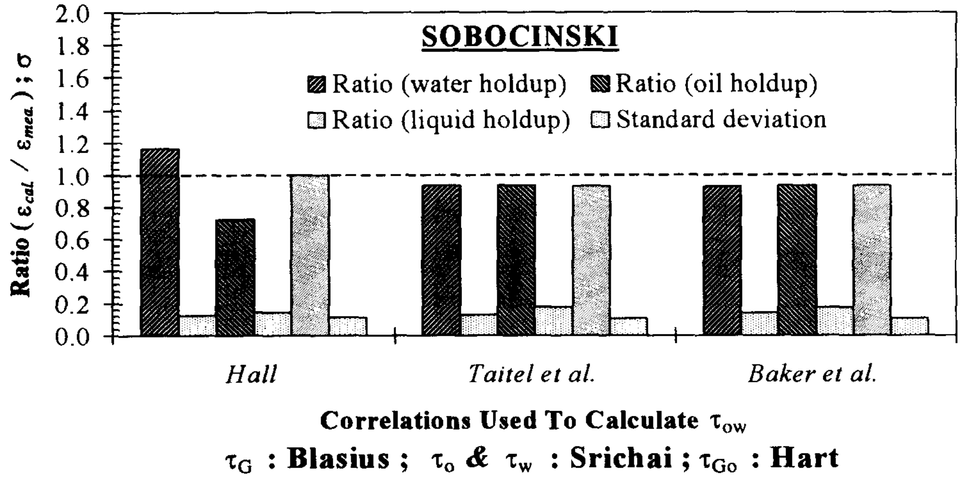

Figure 5. Comparison between correlations used to calculate τ_{ow} (Sobocinski data).

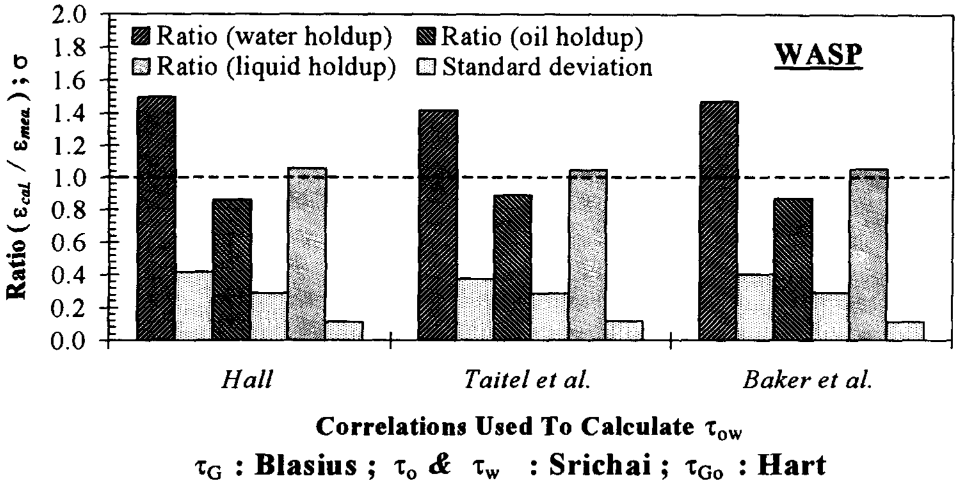

Figure 6. Comparison between correlations used to calculate τ_{ow} (WASP data).

is considered as an opened channel flow between two free moving interfaces and the pipe wall. From a series of comparisons, it is found that when the oil fraction is less than the water fraction, it is better to consider the oil layer as an open channel flow between the free gas–oil interface, and the 'immobile' oil-water interface and the pipe wall. Likewise, when the oil fraction is greater than the water fraction, the oil phase should be taken as a closed channel flow between two interfaces and the pipe wall. In summary, it is concluded that:

(i) when $\epsilon_o < \epsilon_w$:

$$D_o = \frac{4A_o}{(S_o + S_{ow})} \quad [19]$$

(ii) and $\epsilon_o > \epsilon_w$:

$$D_o = \frac{4A_o}{(S_o + S_{ow} + S_{Go})}. \quad [21]$$

However, the switching of the definitions of equivalent diameter dependent on holdup is inconvenient and [19] can be used throughout without great loss of accuracy. This conclusion is supported by the analysis of the Sobocinski and WASP data.

The average ratio of the calculated values to the measured values using the above recommended combination of shear stress correlations and the standard deviation for the two selected sets of data are presented in figure 7. Generally, good prediction is achieved for Sobocinski's low pressure data,

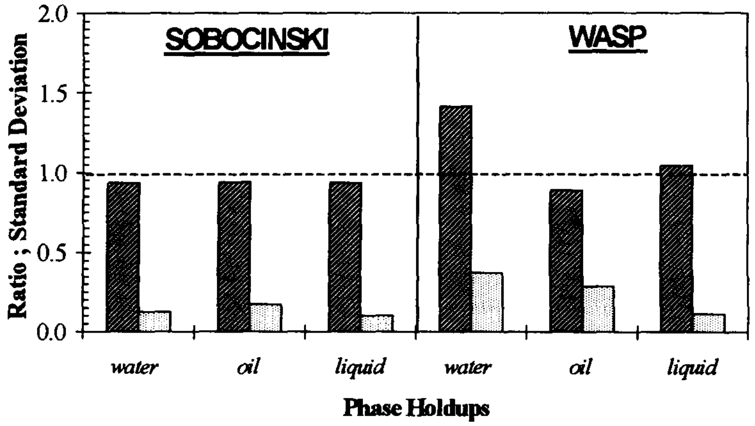

Figure 7. Comparison between the predicted holdups to the measured holdups.

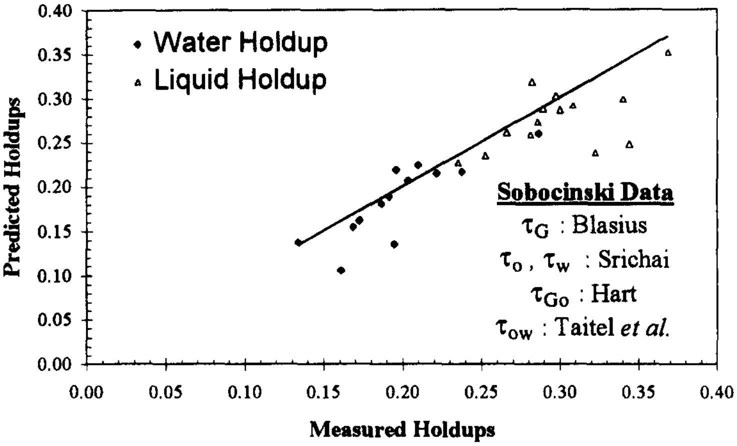

Figure 8. Plot of the predicted water holdup and total liquid holdup to their respective measured values (14 Sobocinski datapoints).

but there is over-prediction in water holdup for the WASP case. Such comparisons for the two sets of experimental data are presented graphically in figures 8 and 9, respectively.

As will be seen, considerable residual errors are present even with the best choice of relationships. In fairness, however, it should be stated that the situation is also true of two-phase flows!

5. CONCLUSIONS

From the comparisons made, the following recommendations are made for the prediction of the respective shear stress terms:

(a) Gas–wall shear stress. Similar predictions were obtained from the simple Blasius (1913) relationship [7] and the Colebrook (1939) relationship [9]. However, in the range of conditions covered (i.e. for Reynolds numbers up to around 240,000), the difference in friction factor is small for the relatively smooth pipe used. The Blasius equation is preferred as it requires less computational time and no information about the pipe roughness. However, this relationship may not apply in the case of very rough pipes.

(b) Oil–wall and water–wall shear stresses. It is recommended that the Srichai (1994) relationships be used here. The relevant friction factor equation is

$$f_{\mathrm{oorw}} = 0.765\,(\epsilon_{\mathrm{oorw}}\mathrm{Re}_{\mathrm{oorw}})^{-0.562}, \qquad [22]$$

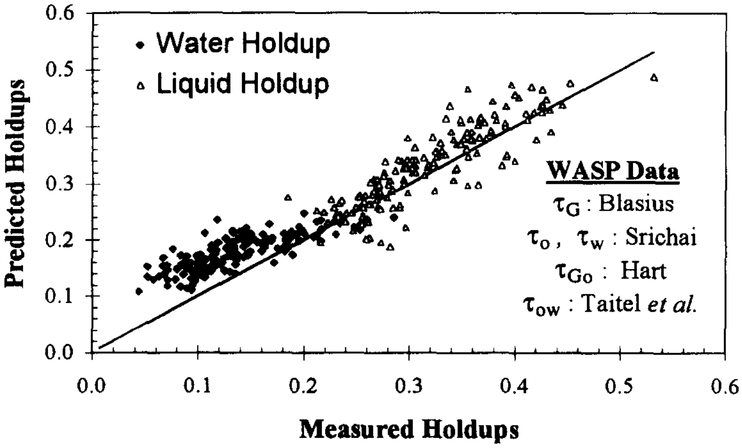

Figure 9. Plot of the predicted water holdup and total liquid holdup to their respective measured values (156 WASP datapoints).

where ϵ_{oorw} is the fraction of the relevant liquid phase and $\mathrm{Re}_{\mathrm{oorw}}$ is the Reynolds number calculated from:

$$\mathrm{Re}_{\mathrm{oorw}} = \frac{u_{\mathrm{oorw}}\rho_{\mathrm{oorw}}D_{\mathrm{oorw}}}{\eta_{\mathrm{oorw}}}. \qquad [23]$$

(c) Gas–oil interfacial shear stress. It is recommended that the relationship of Hart *et al.* (1989) is used; a similar recommendation was made by Spedding *et al.* (1986) in their gas–liquid flow study. The model for evaluating the gas–liquid interfacial friction factor has been modified for three-phase flow case in the present study. The wetted wall fraction, ϕ (for non annular flow) is correlated in terms of the oil phase Froude number

$$\phi = 0.52\epsilon_{\mathrm{L}}^{0.374} + 0.26\mathrm{Fr}_{\mathrm{O}}^{0.58}, \qquad [24]$$

where

$$\mathrm{Fr}_{\mathrm{o}} = \frac{\rho_{\mathrm{o}}}{(\rho_{\mathrm{o}} - \rho_{\mathrm{G}})}\frac{u_0^2}{gD} \qquad [25]$$

$$\epsilon_{\mathrm{L}} = \epsilon_{\mathrm{w}} + \epsilon_{\mathrm{o}}. \qquad [26]$$

The interfacial roughness, k_{i}, is calculated from

$$k_{\mathrm{i}} = 2.3\frac{\epsilon_{\mathrm{L}}D}{4\phi} \qquad [27]$$

and the interfacial friction factor, f_{i} is determined from the explicit form of the Colebrook formula (Eck 1973) as expressed in [10].

(d) Oil–water interfacial shear stress. The straightforward approach which uses a fixed value of f_{ow} of 0.014 as recommended by Taitel *et al.* (1995) is found to give the best prediction.

It should be stated in concluding this paper, that there is, as yet, relatively little quantitative information on three-phase stratified flows. Because of its industrial significance, it is important to produce recommended methods so that these can be used now. However, these methods would be expected to evolve as time goes on and more data is available. An important assumption in the one-dimensional analysis is that the fluids are separate and that the interfaces are (on average) flat. However, an important feature of three-phase stratified flows is inter-entrainment between the fluid layers. This is now being investigated on the Imperial College WASP facility using an isokinetic sampling technique (Khor *et al.* 1996b).

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